

On the electric field screening by electron–positron pairs in the pulsar magnetosphere – II

S. Shibata,¹★ J. Miyazaki² and F. Takahara³

¹*Department of Physics, Yamagata University, 1-4-12 Kojirakawa, Yamagata 990-8560, Japan*

²*Department of Physics, Tokyo Metropolitan University, Minamiohsawa 1-1, Hachioji, Tokyo 192-0397, Japan*

³*Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan*

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ABSTRACT

We study the self-consistency of the pulsar polar cap model, i.e. the problem of whether the field-aligned electric field is screened by electron–positron pairs that are injected beyond the pair production front. We solve the one-dimensional Poisson equation along a magnetic field line, both analytically and numerically, for a given current density incorporating effects of returning positrons, and we obtain the conditions for electric field screening. The formula obtained gives the screening distance and the return flux for given primary current density, field geometry and pair creation rate at the pair production front.

If geometrical screening is not possible, for instance, on field lines with a super-Goldreich–Julian current, then the electric field at the pair production front is constrained to be fairly small in comparison with values typically expected using conventional polar cap models. This is because (1) positive space charge by pair polarization is limited to a small value, and (2) returning positrons leave pair electrons behind. A previous belief that pair creation with a pair density higher than the Goldreich–Julian density immediately screens out the electric field is unjustified, at least for a super-Goldreich–Julian current density. We suggest some possibilities to resolve this difficulty.

Key words: magnetic fields – plasmas – pulsars: general – gamma-rays: theory.

1 INTRODUCTION

One promising model for pulsar action is the polar cap model, in which the field-aligned electric field accelerates charged particles up to TeV energies, and resultant curvature radiation generates an electromagnetic shower. The model can provide an environment for the radio emission mechanism with a particle beam in electron–positron pair plasmas and may also explain the γ -ray pulsation. The pairs will be a particle source of the pulsar wind. The polar cap potential drop is part of the electromotive force produced by the rotating magnetic neutron star. The voltage is a few per cent of the available voltage for young pulsars, while it becomes an important fraction for older pulsars. The localized potential drop is maintained by a pair of anode and cathode regions, the formation mechanism of which is the long-standing issue of the polar cap accelerator.

Because any deviation of the space-charge density from the Goldreich–Julian (GJ) density ρ_{GJ} causes a field-aligned electric field within the light cylinder, how the GJ charge density changes along a given magnetic field line plays an essential role in the formation of the field-aligned electric field. Note here that the GJ density

is determined by the magnetic field geometry: $\rho_{\text{GJ}} \approx -\Omega B_z / 2\pi c$, where Ω is the angular velocity of the star and B_z is the field strength along the rotation axis. For instance, the outer gap, another promising accelerator model, is thought to appear around the null surface on which the GJ density changes its sign. Owing to the field geometry, the outer gap originally has a pair of anode and cathode regions without current and pair plasma. The space-charge-limited flow for the polar cap model on field lines curving towards the rotation axis also has a similar GJ density distribution providing a pair of anode and cathode regions (actually it is regarded as a version of the outer gap model with external current; Hirovani & Shibata 2001).

Once pairs are created in the accelerator, since the pair *number density* far exceeds the GJ value $|\rho_{\text{GJ}}/e|$, where $-e$ is the electronic charge, the dynamics of the created pairs produces crucial effects in localizing the field-aligned electric field.

For the polar cap models, the space-charge-limited flow is found to produce the field-aligned electric field in some cases where the current density and magnetic field geometry are assumed (Fawley, Arons & Scharlemann 1977; Scharlemann, Arons & Fawley 1978). Later on, effects of pair creation and general relativity are taken into account (Arons & Scharlemann 1979; Muslimov & Tsygan 1992). In these models, given a current density, the space-charge density deviates from the GJ charge density, and as a result the field-aligned

★E-mail: shibata@sci.kj.yamagata-u.ac.jp

electric field develops. Since the workfunction of charged particles on the stellar surface is sufficiently small, either electrons or ions can freely escape from the surface, and therefore the electric field on the surface is assumed to be zero. Based on this scheme, primary electrons are shown to be accelerated to energies high enough to induce an electromagnetic shower.

The current density on each magnetic line of force will not be determined by the local electrodynamics in the polar cap, but will be determined in a global manner. Most of the rotation power of the pulsar is conveyed by the pulsar wind, which is accelerated near and beyond the light cylinder. Although the acceleration mechanism of the wind is still a mystery, it is certain that an energetically dominant process which determines the current distribution takes place near and beyond the light cylinder. When the wind is accelerated (or heated), the current runs across the field lines ($\mathbf{E} \cdot \mathbf{j} > 0$) as a simple result of the Poynting theorem, provided that the ideal magneto-hydrodynamics (MHD) condition is more or less valid. Therefore, some of the current coming up from the star crosses the magnetic field lines corresponding to the wind acceleration and returns back to the star. The rest of the current reaches large distances (e.g. nebula shock), and the same amount with opposite direction comes back to the star. In any case, the current closure is seriously controlled by the dynamics near and beyond the light cylinder. The current determined in such a way runs through the polar cap. Thus, the current density distribution across the polar cap magnetic flux should be linked to the outer magnetosphere: the current density distribution is most likely to be determined regardless of the field geometry of the inner magnetosphere. We therefore treat the current density as an adjustable free parameter in our series of papers (Shibata 1991, 1995, 1997; Shibata, Miyazaki & Takahara 1998): the current density is a free parameter for the polar cap model, and it is adjusted when the local model is linked with a model of the outer magnetosphere.

For a space-charge-limited flow, it is shown by Shibata (1997) that both toward and away geometries are possible to generate a large field-aligned electric field causing an electron–positron pair cascade as long as the current density is reasonably large, comparable to the GJ current density $\rho_{\text{GJ}}c$. In his paper, the cases of super-GJ current densities are examined as well as the sub-GJ cases. Note that the GJ current density is simply the GJ current, $I_{\text{GJ}} \approx \mu\Omega^2/c$ divided by the polar cap area, where μ is the magnetic moment of the star. The GJ current is derived by an order-of-magnitude argument, and the net current circulating in the magnetosphere will not be much different from this value. However, if the current is not distributed more or less uniformly over the polar cap, but is focused or fragmented in magnetic flux tubes by analogy with planetary aurorae, then the local current densities can be much higher than the GJ current density. Furthermore, the current density will change in a wide range, field line by field line.

These points motivated us to examine various combinations of current density and magnetic field geometry for the local accelerator. Although deviation of the space charge is found to cause an accelerating electric field, it is not clear how the electric field is screened to complete the localized potential drop. The only known case in which a finite potential drop is formed without pairs is geometrical screening, i.e. the case of toward geometry or general relativistic geometry (Muslimov & Tsygan 1992) with a certain current density that is sub-GJ. For other cases, there remains in general an unscreened electric field in the region where pairs are created. Note that pair plasmas are copious and pair polarization may become efficient. In this paper, we derive the condition for electric field screening immediately behind the pair creation front where an unscreened

electric field is assumed to exist, in the presence of a returning pair. The condition can be used for any current density (super- or sub-GJ) and for both away and toward field line curvatures. We will apply the condition to the cases for which the geometrical effect is not efficient, i.e. away curvature and super-GJ current density. Another important application is the case where pairs are created before the geometrical screening is completed.

In the previous paper (Shibata et al. 1998, hereafter Paper I), we demonstrated that pair polarization does work when enough pairs are injected, where we assumed that pairs are created at a single point; the required multiplication factor is $\sim 10^3$, which is the same order as the value inferred from observations of the Crab pulsar and nebula. However, we suggested that the required multiplicity has to be reached in a very small distance, and this will not be realized in an actual pulsar magnetosphere.

We have made some simplifications in Paper I in order to make the physics as clear as possible: (1) pairs have the same initial Lorentz factor and outward momentum; (2) the pair creation occurs at a single point; and (3) no positrons return to the neutron star. In this paper, we relax the above assumptions (2) and (3) to study more realistic cases: electron–positron pairs are produced continuously along field lines for a finite length, and some particles may stop before reaching the screening point and return to the neutron star surface. As in Paper I, we assume that no frictional force works between the various components. Under these assumptions, we calculate the electric field structure of the pair creation zone and determine the maximum electric field that can be screened under a presence of returning particles. Since the thickness of this zone is geometrically thin, we adopt the one-dimensional approximation in this paper.

In Section 2, we formulate the problem, and in Section 3 we solve the one-dimensional Poisson equation analytically assuming that the pair creation rate is constant as a function of the electrostatic potential. In Section 4, we solve the problem numerically when the pair creation rate is spatially uniform to confirm that the analytic results of Section 3 are sufficiently accurate. In Section 5 we summarize our results and suggest a possible way out to resolve the screening problem.

2 MODEL AND BASIC EQUATIONS

Let us consider a steady model of a field-aligned accelerator that has a finite potential drop with the electric field screened at both ends. For definite sign of charge, let us assume electrons are accelerated outwards, i.e. the electric field points towards the star. Gamma-rays emitted by the electrons convert into pairs beyond a certain surface called the pair production front (PPF); beyond the PPF, pairs are assumed to be created continuously in space.

Pair polarization is an effect that screens out the electric field. The pair positrons are decelerated soon after their birth, while the pair electrons are accelerated, and, as a result of continuity, a positive space charge appears to reduce the electric field. In this paper, we intend to evaluate the space charge produced by pair polarization in the presence of returning positrons.

The notation is the same as in Paper I: the strength of the magnetic field \mathbf{B} is denoted by B , and the component along the rotation axis is B_z ; the non-corotational electric potential Φ gives the field-aligned electric field by $E_{\parallel} = -\mathbf{B} \cdot \nabla\Phi/B$; v and ρ are the velocity and density, with the subscripts ‘1’, ‘+’ and ‘−’ indicating the primary particles, pair positrons and pair electrons, respectively; $\rho_{\text{GJ}} \approx -\Omega B_z/2\pi c$ is the GJ charge density. We also use the following normalized quantities:

$$\begin{aligned}\phi &= \frac{e\Phi}{mc^2}, & j &= \frac{J}{-\Omega B/2\pi}, & j_0 &= \frac{c\rho_{\text{GJ}}}{-\Omega B/2\pi} = \frac{B_z}{B}, \\ \bar{\rho} &= \frac{\rho}{\Omega B/2\pi c}, & \beta &= |v|/c, & \gamma &= (1 - \beta^2)^{-1/2}, \\ x &= \frac{s\omega_p}{c}, & \omega_p &= \sqrt{\frac{4\pi e^2 n_0}{m}}, & n_0 &= \frac{\Omega B}{2\pi e c},\end{aligned}$$

where $J = -en_1v_1$ is the current density of the primary electrons, and s is the coordinate along the magnetic field, which is normalized by the relativistic Debye length.

The Poisson equation for the screening region may be written as

$$-\frac{d^2\phi}{dx^2} = \frac{dE}{dx} = -\frac{j}{\beta_1} + j_0 + \bar{\rho}_+ + \bar{\rho}_-, \quad (1)$$

where $E = -d\phi/dx$ is the normalized electric field along a given magnetic field, and β_1 is the normalized velocity of the primary particles. On the right-hand side of equation (1), the first term represents the negative space charge produced by the primary electronic current stream ($-j/\beta_1 < 0$), and j_0 represents $-\rho_{\text{GJ}}$, which is positive. The difference, $-j/\beta_1 + j_0$, is the space charge that produces the accelerating electric field. The values $\bar{\rho}_{\pm}$ are the normalized charge densities of the pair electrons and pair positrons, which appear beyond the PPF. Note that j and j_0 take positive values in our sign convention.

In a steady acceleration region along given magnetic field lines, ‘an effective space charge’ represented by the right-hand side of (1) becomes positive or negative at each side of the accelerating region as shown in Fig. 1. The negative charge region is located near the stellar surface and electrons are accelerated upwards. In the space-charge-limited flow (Fawley et al. 1977), the negative charge is provided by non-relativistic electrons leaving the stellar surface: $-j/\beta_1 + j_0 < 0$ with $\beta_1 \ll 1$. Additional negative charge is induced on the side wall of the flow tube. As the electrons are accelerated, $\beta_1 \rightarrow 1$, the effective space charge becomes positive above a certain height, $-j + j_0 > 0$, if j is smaller than j_0 . Thus, the anode region is formed in this case.

Further development is to include the effect of magnetic field geometry: on field lines curving towards the rotation axis, $j_0 = B_z/B$ (< 1) increases along the field lines while $j \propto J/B$ is constant, and therefore even if $-j + j_0 < 0$ near the star, as the flow goes out,

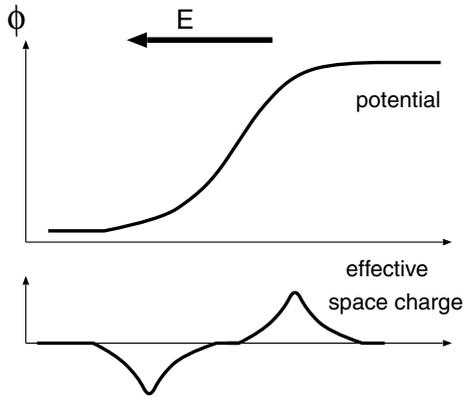


Figure 1. An accelerating region with a finite potential drop has a pair of anode and cathode regions. For a parallel (not antiparallel) rotator, negative space charge appears in the side of the stellar surface, and positive space charge is required for ‘screening’ the electric field at the other side of the potential drop. The positive screening charge can be provided by toward curvature (on field lines curving towards the rotation axis) and by a pair plasma.

$-j + j_0$ can be positive above some distance as far as $j < 1$ (sub-GJ). This toward curvature is efficient to screen the electric field which is sufficiently strong to cause pair cascade.

One can choose a particular value of j so that the electric field is localized with the screened electric field at both ends without pairs. However, as has been mentioned, the current density is determined in a global way. It is therefore unlikely that such a particular value of the current density is realized in general. One can expect that the PPF is formed at the place where the electric field remains unscreened.

If $j > 1$, the anode formation by the geometrical effect does not work because $-j + j_0 < 0$. The same situation takes place for any value of j on field lines curving away from the rotation axis (Mestel 1981). In these cases, it is possible for the steady accelerator to exist only if an anode region is provided in some way, for which we study the dynamics of pairs in the PPF region.

Since screening by pairs occurs in a small scale such as $\gamma mc^2/e|E_{\parallel}|$, j and j_0 are assumed to be constant in the screening region around the PPF.

3 CONDITION FOR SCREENING

The screening region is schematically shown in Fig. 2. The pair creation starts at the PPF ($x = 0$), and continues beyond ($x \geq 0$). The non-dimensional electric field at $x = 0$ is denoted by $-E_r$, and the potential at the PPF is used for reference, i.e. $\phi = 0$ at $x = 0$. Let us assume that the field-aligned electric field is screened out at a certain point defining the screening surface (SS).

Multiplying equation (1) by E and integrating it from the PPF to the SS, we obtain the screening condition

$$\frac{1}{2}E_r^2 = \int_0^{\phi_s} (\bar{\rho}_+ + \bar{\rho}_-) d\phi - (j - j_0)\phi_s, \quad (2)$$

where we have assumed for the primary particles $\beta_1 = 1$, and the subscript ‘s’ indicates values at the SS. This condition is equivalent to the column charge density between the PPF and the SS being equal to E_r (Gauss’s law).

The net pair flux produced between the PPF and a position x is given by the multiplication factor, $m(x)$, times the primary electron flux, j . The multiplication factor $m(x)$ is obviously a monotonic function of x . We assume that ϕ is also a monotonic function of x between the PPF and the SS. Thus either x , m or ϕ can be used to designate the coordinate position.

The charge densities of pairs produced by a small flux element between m and $m + dm$ at a ‘position ϕ ’ are given by

$$d\bar{\rho}_{\pm} = \pm j \frac{dm}{\beta_{\pm}} = \pm \frac{j\gamma_{\pm}}{\sqrt{\gamma_{\pm}^2 - 1}} dm, \quad (3)$$

where the Lorentz factors γ_{\pm} are measured at the position ϕ and are functions of the injection point denoted by $\phi(m)$, i.e.

$$\gamma_- = \phi - \phi(m) + \bar{\gamma} \quad (4)$$

for electrons and

$$\gamma_+ = -\phi + \phi(m) + \bar{\gamma} \quad (5)$$

for positrons; $\bar{\gamma}$ is the Lorentz factor at creation, and is assumed to be a universal constant, say $\bar{\gamma} \sim 10^2$.

Positrons return back to the star when

$$\phi_{\text{ret}}(m) \equiv \phi(m) + \bar{\gamma} - 1 < \phi_s, \quad (6)$$

where $\phi_{\text{ret}}(m)$ indicates the returning ‘position’, otherwise positrons have enough energy to go through the SS. Thus there is a separatrix M_1 below which the positron flux ($0 < m < M_1$) returns and

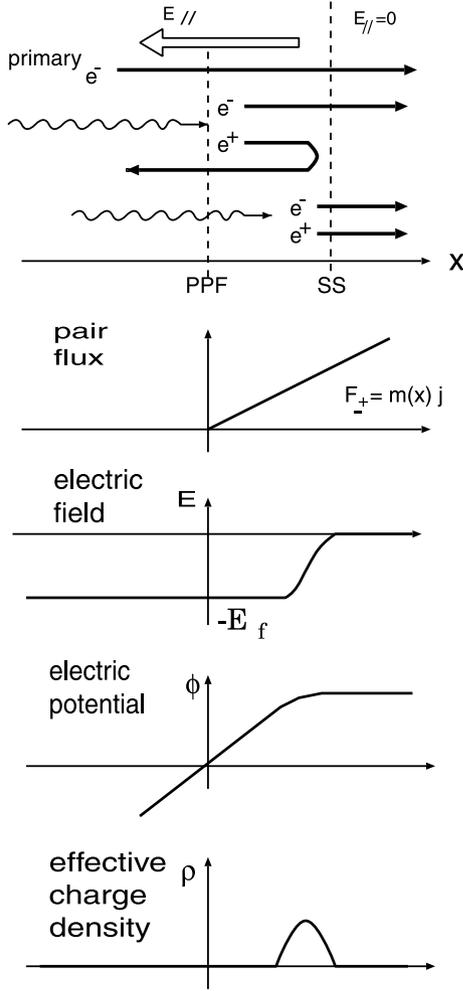


Figure 2. Schematic diagram of the screening region by pair polarization. Electron–positron pair plasma is created at more or less uniform rate beyond the pair production front (PPF). The pair positrons are decelerated while the pair electrons are accelerated. When the pair positrons become non-relativistic, there appears positive space charge, which should screen out the accelerating electric field. Some of the pair positrons will turn back to the star. Pairs created near the screening surface (SS) go through the region.

beyond which the positron flux ($M_1 < m < M_2$) passes through the SS, where M_2 is the multiplication factor at the SS.

The potential at which the flux M_1 is created is given by the marginal condition of equation (6)

$$\phi_1 \equiv \phi(M_1) = \phi_s - \bar{\gamma} + 1. \quad (7)$$

If there are returning positrons, the space charge before the PPF becomes $-(j/\beta_+) + j_0 + jM_1$, so that a positive contribution is added by the returning flux. For acceleration of the primary electrons, the column charge density between the star surface and the PPF must be negative. If too many positrons are reflected back, the negative column density below the PPF will be broken down. If M_1 is given and is small enough to assure the negativeness of the column charge below the PPF, the growth of the electric field before the PPF is calculated for an assumed j as in the conventional space-charge-limited flow.

Once the primary particles become relativistic, the space charge before the PPF is represented by $-j + j_0 + jM_1$. If this space charge is negative, then we will have further acceleration, while if it is

positive, then we have screening. We may have $-j + j_0 + jM_1 > 0$, for instance, on field lines curving towards the rotation axis. The positiveness does not guarantee vanishing of E_{\parallel} at the PPF because the PPF may be formed at a place where E_{\parallel} still remains unscreened. For this case, the present analysis gives the screening distance after the PPF. On the other hand, the PPF can be formed in a negative space-charge region where

$$-j + j_0 + jM_1 \leq 0. \quad (8)$$

This takes place on field lines with away curvature or with super-GJ current densities (for any type of field line curvatures). For this case, screening relies on pair polarization.

It is notable that the observed X-ray flux from the heated polar cap suggests that M_1 is much less than unity.

In addition to the screening condition (2), we have a supplementary condition that the second derivative of ϕ must not be positive on the SS: because $d\phi/dx$ is positive at $x=0$ and zero at the SS, if $d^2\phi/dx^2$ were positive at the SS, $d\phi/dx$ would be negative just below the SS, indicating that $d\phi/dx$ vanishes inside the SS, which contradicts the definition of the SS. Integration of (3) up to M_2 yields the pair charge density at the SS, so that we have positiveness of the effective space-charge density at the SS as

$$\rho_s = -j \int_0^{M_2} \frac{dm}{\beta_-} + j \int_{M_1}^{M_2} \frac{dm}{\beta_+} - (j - j_0) \geq 0. \quad (9)$$

With the help of (3), the contribution of pairs to the ‘column charge’ in the ϕ coordinate is given by

$$\begin{aligned} & \int_0^{\phi_s} (\bar{\rho}_+ + \bar{\rho}_-) d\phi \\ &= \int_0^{M_1} j dm \int_{\phi(m)}^{\phi_{\text{ret}}(m)} \frac{d\phi}{\beta_+} + \int_0^{M_1} j dm \int_0^{\phi_{\text{ret}}(m)} \frac{d\phi}{\beta_+} \\ & \quad - \int_0^{M_1} j dm \int_{\phi(m)}^{\phi_s} \frac{d\phi}{\beta_-} + \int_{M_1}^{M_2} j dm \int_{\phi(m)}^{\phi_s} \left(\frac{1}{\beta_+} - \frac{1}{\beta_-} \right) d\phi. \end{aligned} \quad (10)$$

In the above equation, the first term comes from the up-going positrons which eventually stop at ϕ_{ret} . The second term is due to the returning positrons. The third term is for the electrons that are paired with the reflected positrons. The fourth term is for the pairs that pass through the region.

The first term is integrated exactly to yield

$$\begin{aligned} \left(\text{1st term} \right) \\ \left(e^+ \rightarrow \right) &= \int_0^{M_1} j dm \int_{\phi(m)}^{\phi_{\text{ret}}(m)} \frac{\gamma_+}{\sqrt{\gamma_+^2 - 1}} d\phi \\ &= jM_1 \sqrt{\bar{\gamma}^2 - 1} \approx jM_1 \bar{\gamma} - jM_1 \Delta_1, \end{aligned} \quad (11)$$

where the last expression is obtained if $\bar{\gamma} \gg 1$, and $\Delta_1 = (2\bar{\gamma})^{-1} \ll 1$.

The second term becomes

$$\begin{aligned} \left(\text{2nd term} \right) \\ \left(\leftarrow e^+ \right) &= j \int_0^{M_1} j dm \sqrt{[\bar{\gamma} + \phi(m)]^2 - 1}, \\ &\approx j \int_0^{M_1} dm \left\{ [\bar{\gamma} + \phi(m)] - \frac{1}{2[\bar{\gamma} + \phi(m)]} \right\} \\ &= jM_1 \bar{\gamma} + j \int_0^{M_1} \phi(m) dm - jM_1 \Delta_2 \end{aligned} \quad (12)$$

with

$$\Delta_2 = \left\langle \frac{1}{2[\bar{\gamma} + \phi(m)]} \right\rangle_0^{M_1} \ll 1, \quad (13)$$

where $\langle f(m) \rangle_{m_1}^{m_2}$ indicates an average with respect to the flux m , defined as

$$\langle f(m) \rangle_{m_1}^{m_2} = \frac{1}{m_2 - m_1} \int_{m_1}^{m_2} dm f(m).$$

The third term of (10) is the column charge by electrons associated with the returning positrons:

$$\begin{aligned} \left(\begin{array}{l} \text{3rd term} \\ e^- \rightarrow \end{array} \right) &= -j \int_0^{M_1} dm \left\{ \sqrt{[\bar{\gamma} + \phi_s - \phi(m)]^2 - 1} \right. \\ &\quad \left. - \sqrt{\bar{\gamma}^2 - 1} \right\} \\ &\approx -j \int_0^{M_1} [\phi_s - \phi(m)] dm + j M_1 \Delta_3 \end{aligned} \quad (14)$$

with

$$\Delta_3 = \left\langle \frac{1}{2\gamma_{-s}} \right\rangle_0^{M_1} - \frac{1}{2\bar{\gamma}} \ll 1, \quad (15)$$

where $\gamma_{-s}(m) \equiv \bar{\gamma} + \phi_s - \phi(m)$ is the Lorentz factor of pair electrons at the SS.

The fourth term becomes

$$\begin{aligned} \left(\begin{array}{l} \text{4th term} \\ e^\pm \rightarrow \end{array} \right) &= j \int_{M_1}^{M_2} dm \left(2\sqrt{\bar{\gamma}^2 - 1} - \sqrt{\gamma_{-s}^2 - 1} \right. \\ &\quad \left. - \sqrt{\gamma_{+s}^2 - 1} \right), \end{aligned} \quad (16)$$

where $\gamma_{+s}(m) \equiv \bar{\gamma} - \phi_s + \phi(m)$ is the Lorentz factor of the positrons at the SS. If one assumed $\bar{\gamma}$, γ_{-s} and $\gamma_{+s} \gg 1$, then the integrand would be $\bar{\gamma} - \gamma_{-s} + \bar{\gamma} - \gamma_{+s}$ which would vanish after inserting the definition of γ_{-s} and γ_{+s} . This means simply that pairs passing through with relativistic speeds would not cause any space charge. However, the positron flux near M_1 causes positive net charge because $\gamma_{+s} \sim 1$ near $\phi \sim \phi_s$, so that the fourth term cannot be ignored: we rearrange (16) as

$$\begin{aligned} \left(\begin{array}{l} \text{4th term} \\ e^\pm \rightarrow \end{array} \right) &= 2j \int_{M_1}^{M_2} \left(\sqrt{\bar{\gamma}^2 - 1} - \bar{\gamma} \right) dm \\ &\quad - j \int_{M_1}^{M_2} \left(\sqrt{\gamma_{+s}^2 - 1} - \gamma_{+s} \right) dm \\ &\quad - j \int_{M_1}^{M_2} \left(\sqrt{\gamma_{-s}^2 - 1} - \gamma_{-s} \right) dm, \end{aligned} \quad (17)$$

which can be integrated exactly (see Appendix) if the pair creation rate is constant with respect to the ‘ ϕ coordinate’. In this approximation, we have

$$m = M_1 + \alpha_\phi [\phi(m) - \phi_1] \quad (18)$$

with

$$\alpha_\phi = \frac{M_2 - M_1}{\phi_s - \phi_1} = \frac{M_2 - M_1}{\bar{\gamma} - 1} = \text{constant}, \quad (19)$$

where α_ϕ is the pair multiplication factor created in a unit potential drop mc^2/e , in other words, α_ϕ is the pair multiplication factor produced in the ‘braking distance’ $\Delta s = mc^2/e|E_{\parallel}|$. After integration we have

$$\left(\begin{array}{l} \text{4th term} \\ e^\pm \rightarrow \end{array} \right) = j\alpha_\phi \zeta, \quad (20)$$

where ζ is a constant of the order of unity; $\zeta = 1.744$ for $\bar{\gamma} = 100$ (see Appendix for derivation).

Near the SS, $d\phi/dx \rightarrow 0$, $\alpha_\phi = dm/d\phi$ is not constant if the pair creation is spatially uniform. In the next section, we perform a nu-

merical calculation for spatially uniform pair creation and find that the above evaluation is quite accurate. The analytical evaluation by using a constant rate $\alpha_\phi = dm/d\phi$ will be appropriate for more or less uniform pair creation.

Inserting the above evaluation of all the four terms into (2) and rearranging relevant terms, we have

$$\frac{1}{2} E_f^2 = 2j M_1 - 2j M_1 \langle \Delta\phi \rangle + j\alpha_\phi \zeta - (j - j_0 - j M_1) \phi_s, \quad (21)$$

where

$$\begin{aligned} \langle \Delta\phi \rangle &= \frac{1}{M_1} \int_0^{M_1} [\phi_s - \phi_{\text{ret}}(m)] dm \\ &= \frac{1}{M_1} \int_0^{M_1} [\phi_1 - \phi(m)] dm \end{aligned} \quad (22)$$

is the potential drop (flux-weighted) in the region where returning lasts – in other words, the potential difference between where returning starts and where screening completes.

The pair effect is made up of three components corresponding to the first three terms in equation (21). The first effect is a positive charge produced by the non-relativistic part of the returning positrons. The second effect is negative because it is due to electrons left behind the returning positrons. This effect by returning is fatal: the more positrons that return, the more electrons that are left behind to create negative space charge in the downstream (outer) region, whereas the region needs positive charge for screening. The last effect is caused by polarization of the pairs that pass through the SS (third term).

If the region of returning is large, in other words, if $\langle \Delta\phi \rangle \gg 1$, the pair screening becomes obviously difficult to complete because the negative space charge produced by pair electrons left behind in the upstream region becomes large to prevent screening. If uniform pair creation is assumed, $\langle \Delta\phi \rangle = \frac{1}{2} \phi_1$ and $M_1 = \alpha_\phi \phi_1$. In this case, the condition (21) becomes

$$\frac{1}{2} E_f^2 = j\alpha_\phi [\phi_1(2 - \phi_1) + \zeta] - (j - j_0 - j M_1) \phi_s. \quad (23)$$

The first term of the right-hand side of (23) corresponds to the pair polarization. The second term is the ‘background charge’, the sign of which depends on the global parameters j and j_0 , and where the PPF is formed. Let us restrict ourselves to the case for which the global effect is not helpful but pairs cause screening, i.e. the last term makes no positive contribution. In order for the right-hand side of (23) to be positive, the necessary condition is that the pair polarization term is positive, and then we have

$$2\langle \Delta\phi \rangle = \phi_1 < 1 + \sqrt{1 + \zeta} \approx 2.6, \quad (24)$$

which means that returning of positrons lasts only a few braking distances.

Here we invoke the condition (9) for charge density at the SS to be positive:

$$\begin{aligned} \rho_s &= -j \int_0^{M_2} \frac{dm}{\beta_-} + j \int_{M_1}^{M_2} \frac{dm}{\beta_+} - (j - j_0) \\ &= j\alpha_\phi \left[2\sqrt{\bar{\gamma}^2 - 1} - \sqrt{(2\bar{\gamma} + \phi_1 - 1)^2 - 1} \right] - (j - j_0) \\ &\approx (1 - \phi_1)\alpha_\phi j - (j - j_0) \\ &= j\alpha_\phi(1 - 2\phi_1) - (j - j_0 - M_1 j) > 0. \end{aligned} \quad (25)$$

As was mentioned, returning positrons leave negative space charge at the SS, the positiveness of ρ_s sets a strong constraint: from (25), again noting no positive contribution from the last term, we have $(1 - 2\phi_1) > 0$, i.e.

$$0 < \phi_1 < \frac{1}{2}. \quad (26)$$

The condition (24) is then rewritten as

$$\frac{1}{2}E_f^2 = j\alpha_\phi\zeta' - (j - j_0 - jM_1)\phi_s, \quad (27)$$

where ζ' is between $\zeta = 1.744$ (when $\phi_1 = 0$) and $\zeta + \frac{3}{4} = 2.494$ (when $\phi_1 = \frac{1}{2}$). Eventually the necessary condition for pair polarization to screen out E_f becomes

$$\alpha_\phi \geq \frac{E_f^2}{2j\zeta'}. \quad (28)$$

Recall that α_ϕ is the pair multiplication factor produced *within the braking distance* $\Delta s = mc^2/e|E_\parallel|$. In dimensional form, the required pair multiplication factor in Δs is given by

$$\Delta M_{\text{screen}} \geq \frac{E_\parallel^2}{8\pi mc^2 n_0 \zeta' j}. \quad (29)$$

For polar cap models, typical values of the right-hand side of (29) are as large as 10^3 . For instance, given a primary acceleration of $\gamma \sim 10^6$ in a distance of $L \sim 10^4$ cm, we may have $|E_\parallel| = 1.7 \times 10^5$ esu or $E_\parallel^2/8\pi = 1.2 \times 10^9 (\gamma_6/L_4)^2$ erg cm $^{-3}$ at the PPF, and

$$\Delta M_{\text{screen}} \geq 2.0 \times 10^3 \left(\frac{\gamma_6}{L_4}\right)^2 \left(\frac{P_{0.1}}{B_{12}}\right) \frac{1}{j\zeta'}, \quad (30)$$

where $\gamma_6 = \gamma/10^6$, and L_4 , $P_{0.1}$ and B_{12} are in units of 10^4 cm, 0.1 s and 10^{12} G, respectively. This multiplication factor should be achieved within a distance of $\Delta s = mc^2/e|E_\parallel| = \gamma^{-1}L \approx 10^{-2}$ cm for screening. It seems quite difficult that the pair multiplication factor should be of the order of 10^3 in this sort of small distance. If the pair production takes place over a dimension of say 10^4 – 10^5 cm, then the required multiplication factor in the whole region is 10^9 – 10^{10} and is much higher than the value predicted by conventional magnetic pair production. If, on the other hand, we assume $\approx 10^6$ cm, then $\Delta M_{\text{screen}} \approx 0.2$, $\Delta s = 1$ cm and the total multiplication factor becomes 2000 to 20 000, which may be allowed.

In the case where $j < j_0$, the screening is completed with the help of the ‘background’ positive space charge, i.e. $-(j - j_0 - jM_1)\phi_s$ in equation (23). However, the distance to the screening region is enlarged by the negative contribution due to the pair electrons left behind. For a given PPF, i.e. given j , j_0 and pair creation rate α_ϕ , we can calculate the screening distance in potential scale ϕ_s and return flux M_1 in use of (23), (7) and $M_1 = \alpha_\phi\phi_1$:

$$\phi_s = \frac{\frac{1}{2}E_f^2 + j\alpha_\phi(\bar{\gamma}^2 - 1 - \zeta)}{j_0 - j[1 - \alpha_\phi(\bar{\gamma} + 1)]}. \quad (31)$$

It can be seen that screening is possible if

$$j < j_0/[1 - \alpha_\phi(\bar{\gamma} + 1)] \equiv j_{\text{cr}}.$$

For the conventional polar cap model, $j_{\text{cr}} \approx j_0$ because $\alpha_\phi\bar{\gamma}$ is much less than unity. If $j \rightarrow j_{\text{cr}}$, M_1 becomes large so that we have to treat the whole acceleration region including the cathode region, which will be discussed in a subsequent paper.

4 NUMERICAL RESULTS

In the previous section, we have discussed the screening condition under the assumption that the pair creation rate is constant with respect to the electrostatic potential. In this section, we present numerical solutions when the pair creation rate is spatially uniform, $dm/dx = \text{constant}$.

First we check the code by calculating the same case as in the previous section. We present the case of $\bar{\gamma} = 100$, $j = 2j_0$, $\alpha_\phi = 1$ and $M_1 = 0.5$, and hence $\phi_1 = 0.5$. The results are shown in Figs 3 and 4, where ϕ and $|E|$ as functions of x are shown. We find that the screening of the electric field occurs in the region where non-

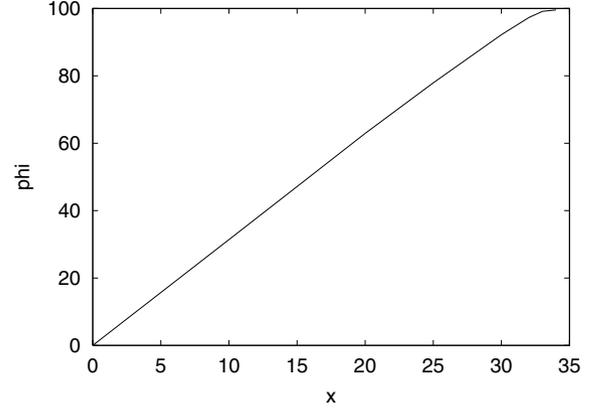


Figure 3. Electrostatic potential as a function of the distance from the pair production front.

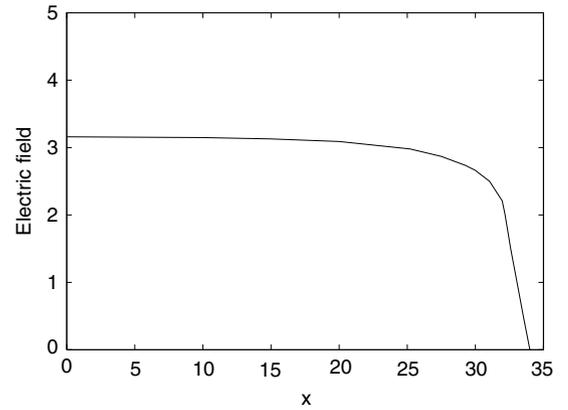


Figure 4. Electric field strength as a function of the distance from the pair production front.

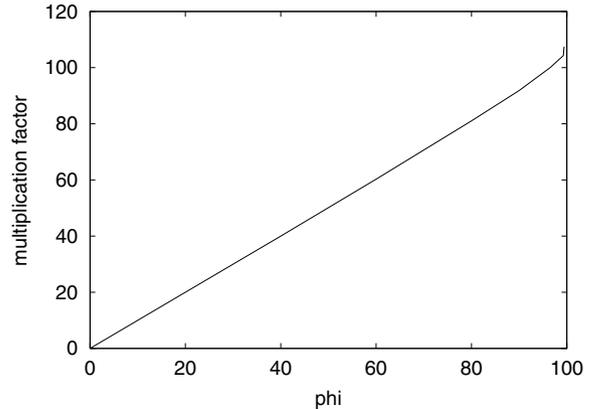


Figure 5. Multiplication factor as a function of ϕ when dm/dx is constant.

relativistic positrons appear, and the distance scale of this region is roughly the braking scale of $mc^2/e|E_\parallel|$. The whole dimension between the PPF and the SS is $\bar{\gamma}$ in potential scale. Consequently 99 per cent of the pairs produced in this region go through and only 1 per cent of positrons return back to the star. In this case, the numerical calculation gives $E_f = 3.159$, which agrees perfectly with the numerical estimate based on condition (28),

Next, we examine the case where the pair creation rate is spatially constant, i.e. $m(x) = \alpha_x x$ with a constant α_x . For $\bar{\gamma} = 100$, $j = 2j_0$ and $M_1 = 0.5$, we solve equation (1) numerically. When $\phi_1 = 0.5$,

we obtain the electric field strength at the PPF, $E_f = 3.160$, which is almost the same value as the case of constant α_ϕ . If one plots the same diagram as in Figs 3 and 4, it is hard to find a difference in the plots.

The small difference found in E_f between the two cases arises from the behaviour of the pair creation rate near $\phi = \phi_s$. When we assume $m = \alpha_\phi \phi$, the spatial pair creation rate becomes slightly smaller as the electric field becomes weaker near the SS. With this numerical solution, we find that (28) is useful even for the case of spatially uniform pair creation. Fig. 5 shows the multiplication factor, the slope of which increases very near the SS.

5 CONCLUDING REMARKS

In this paper, we considered the electric field screening by polarization of electron–positron pairs which are created beyond the pair production front as is generally supposed for the polar cap models. We allow the pair positrons to return back to the star as long as the condition for the acceleration is satisfied. The one-dimensional Poisson equation is solved together with particle motion for a steady-state acceleration region. We obtain the formula (23) which gives the electric field strength that can be screened for a given current density j , a given field geometry j_0 and a given pair creation rate α_ϕ . It is found that (1) pair polarization has little contribution for positive space charge, and (2) returning positrons make the screening seriously difficult because the pair electrons left behind the returning positrons produce negative space charge in the screening region where positive space charge is required. As a result, the thickness of the screening is restricted to be as small as the braking distance $\Delta s \approx mc^2/e|E_\parallel|$ for which positrons become non-relativistic. We confirmed the previous result of Paper I that the electric field in the case where geometrical screening is not possible, i.e. $-j + j_0 + jM_1 < 0$ at the pair production front, is given, from (29), by

$$\frac{E_\parallel^2}{8\pi} < mc^2 \left(\frac{\Omega B}{2\pi ce} \right) \zeta' j \Delta M_{\text{screen}}, \quad (32)$$

where ΔM_{screen} is the pair multiplication factor *within* Δs . If the primary current density is of the order of the Goldreich–Julian (GJ) value, the required pair multiplication factor per primary electron is enormously large and cannot be realized in conventional pair creation models. A previous belief that pair creation with a pair density higher than the GJ density immediately screens out the electric field is unjustified.

Some mechanism to salvage this difficulty should be found. As already mentioned, if the last term of (23), $-(j - j_0 - jM_1)\phi_s$, becomes positive, screening is possible as long as $j < j_{\text{cr}} \approx j_0$. Since $j_0 = B_z/B < 1$, j must be less than unity. It must be noted that a positive space charge exists before the pair production front in this case, so that screening takes place before the pair production front and E_\parallel is reduced. Even a reversal of E_\parallel is possible for a small j (Shibata 1995). In any case, if geometrical screening works, a finite potential drop along field lines can complete. But the current density is restricted in a certain range.

The condition (29) implies that, if the current density is much higher than the GJ value, the required multiplication factor can be reduced. Although the total current circulating in the magnetosphere should be of the order of the so-called GJ current, $I_{\text{GJ}} = \mu\Omega^2/c$, where μ is the magnetic moment of the star, current densities can in principle be focused: the GJ current density is just a value if I_{GJ} is more or less uniformly distributed over the whole polar cap area, and therefore if the current is distributed, say, in an annular

region with fine structures such as seen in aurora, the focused current density will be much higher than the GJ current density. Thus, if j is much larger than j_0 , the required multiplication factor will be reduced. [If the current is super-GJ, toward curvature will not be helpful for screening because $-(j - j_0 - jM_1)$ is negative.] In any case, the physics in focused field-aligned current, e.g. effects of inhomogeneity across the magnetic field lines, and two stream instability in a high current density with unscreened electric field, has not yet been properly studied.

Since $\bar{\gamma}$ is of the order of 100, ϕ_s turns out to be of the order of 100 using (7) and (26). From (19) we may safely approximate as $\alpha_\phi = M_2/100$. From (28), $E_f \leq \sqrt{M_2}/5$, taking $\zeta' = 2$ and $j = 1$. As an example, let us assume that total pair production occurs with a multiplication factor 10^4 over the distance $x_p = 10^4$. Thus, $M_2 = 10^4(x_s/x_p) = x_s$. Noting that $E_f \approx \phi_s/x_s = 100/x_s$, we obtain $x_s > 60$ and $E_f < 1.5$. Since the unit length c/ω_p is typically 1 cm, this electric field corresponds to a voltage mc^2/e per cm. Assuming that primary electrons are accelerated up to $\gamma > 10^6$ with this weak field $E_f \sim 1$, the required length between the neutron star surface and the pair production front is about 10^6 cm in height. Thus pair screening is possible if the pair production front is located high above the surface and if the field-aligned electric field is not so strong. However, the previous models predict a stronger electric field, so we would need an additional ingredient to keep a small field as long as a stellar radius. In any case, a weak electric field kept in a long distance is one possible solution to have a self-consistent polar cap model.

Another possible way out is to include frictional forces between various components of charged currents. Since friction on positrons pulls them outwards along with electrons, the returning fraction of positrons will be reduced, which will make screening easier. Although there are some ideas for physical processes of friction (two stream instability, production of positroniums and others), it is not clear whether the frictional force can be strong enough to lift positrons and screen the electric field. Study of the frictional interaction under an *unscreened* electric field is urgently required.

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REFERENCES

- Arons J., Scharlemann E. T., 1979, ApJ, 231, 854
- Fawley W. M., Arons J., Scharlemann E. T., 1977, ApJ, 217, 227
- Hirofani S., Shibata S., 2001, ApJ, 558, 216
- Mestel L., 1981, in Sieber W., Wielebinski R., eds, Proc. IAU Symp. 95, Pulsars: 13 Years of Research on Neutron Stars. D Reidel, Dordrecht, p. 9
- Muslimov G. A., Tsygan A. I., 1992, MNRAS, 225, 61
- Scharlemann E. T., Arons J., Fawley W. M., 1978, ApJ, 222, 297
- Shibata S., 1991, ApJ, 378, 239
- Shibata S., 1995, MNRAS, 276, 537
- Shibata S., 1997, MNRAS, 287, 262
- Shibata S., Miyazaki J., Takahara F., 1998, MNRAS, 295, L53 (Paper I)

APPENDIX A: SPACE CHARGE BY PAIRS GOING THROUGH THE SCREENING REGION

The space charge by pairs that go through the screening region is given by (17). The second and third terms can be integrated analytically if the pair flux is proportional to the potential for which

$$dm = \alpha_\phi d\phi(m) \quad (\text{A1})$$

where the pair creation rate in unit potential drop α_ϕ is assumed to be constant.

The second term of (17) becomes

$$\begin{aligned} & j \int_{M_1}^{M_2} dm \left(\gamma_{+s} - \sqrt{\gamma_{+s}^2 - 1} \right) \\ &= j \int_{\phi_1}^{\phi_s} \left(\gamma_{+s} - \sqrt{\gamma_{+s}^2 - 1} \right) \alpha_\phi d\phi(m) \\ &= \frac{1}{2} j \alpha_\phi \left[\bar{\gamma}^2 - \bar{\gamma} \sqrt{\bar{\gamma}^2 - 1} + \ln \left(\bar{\gamma} + \sqrt{\bar{\gamma}^2 - 1} \right) - 1 \right]. \quad (\text{A2}) \end{aligned}$$

In the same way, the third term of (17) becomes

$$\begin{aligned} & j \int_{M_1}^{M_2} dm \left(\gamma_{-s} - \sqrt{\gamma_{-s}^2 - 1} \right) \\ &= j \int_{\phi_1}^{\phi_s} \left(\gamma_{-s} - \sqrt{\gamma_{-s}^2 - 1} \right) \alpha_\phi d\phi(m) \\ &= \frac{1}{2} j \alpha_\phi \left[\gamma_{-s}^2 - \gamma_{-s} \sqrt{\gamma_{-s}^2 - 1} \right. \\ &\quad \left. + \ln \left(\gamma_{-s} + \sqrt{\gamma_{-s}^2 - 1} \right) - 1 \right]_{\bar{\gamma}}^{2\bar{\gamma}-1}. \quad (\text{A3}) \end{aligned}$$

Thus we arrive at (20) with $\zeta = 1.744$ if $\bar{\gamma} = 100$ is substituted.

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